J. Adhesion, 1973, Vol. 5, pp. 81–87

Printed in Northern Ireland

Note

Cohesive-Adhesive Fracture in a Pressurized Double Blister

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(*Received March* 9, 1972)

If two infinite sheets of different elastic properties are bonded together by an adhesive material of even different elastic properties except for an unbonded strip of width $2a$ into which pressure is introduced, adhesive fracture can occur by unbonding between the adhesive and either of the two sheets, of cohesive fracture can arise from an unstable flaw within the adhesive. This paper describes an approximate analysis through which the critical applied pressure and preferred locus of fracture initiation can be estimated as a function of the geometrical and mechanical properties of the three layers involved.

INTRODUCTION

In previous papers,[1,2] the critical applied pressure, $p_{0_{cr}}$, to debond a thin layer of an incompressible medium from a rigid substrate was calculated approximately by utilizing beam theory and the interchange of stored strain energy with that required to create new surface area. Two cases were considered: (1) the layer cast and bonded directly upon the substrate,[1] and (2) the layer pre-cured and bonded to the substrate by an intermediate adhesive layer.[3] The result for an infinite strip is†

$$p_{0_{cr}}^2 = \frac{2\left(\dfrac{h}{a}\right)^3\left(\dfrac{E\gamma_a}{a}\right)}{1 + \dfrac{16\mu^4 + 56\mu^3 + 84\mu^2 + 63\mu + 18}{4\mu^3(1+\mu)^2}} \equiv \frac{2\left(\dfrac{h}{a}\right)^3\left(\dfrac{E\gamma_a}{a}\right)}{1 + \phi(\mu)} \qquad (1)$$

† In Reference 2, Eq. (14) is improperly described as the result for a plate strip. It is actually that for a beam strip of unit width. The plate strip solution is derived from the beam strip one by replacing E by $E/(1 - \nu^2)$, which latter value for an incompressible material is $4E/3$. Replacing E by $4E/3$ in that Eq. (14) gives the result in (1) above.

in which E is the tensile modulus, γ_a is the specific adhesive fracture energy, $2a$ is the unbonded length of the strip and h is the strip thickness. The parameter μ reflects the correction due to a finite adhesive layer thickness (h') with material properties (E', ν') and a foundation, or spring, modulus (k) in the Winkler sense such that

$$\mu \equiv \lambda a; \quad 4\lambda^4 \equiv k/D \tag{2}$$

in which D is the flexural modulus $Eh^3/12(1 - \nu^2)$ of the strip. If the layer thickness is laterally restrained in its plane ($\varepsilon_x = \varepsilon_y = 0$), then

$$k = \frac{(1 - \nu')(E/h')}{(1 - 2\nu')(1 + \nu^2)} \quad \text{(one-sided strip)} \tag{3}$$

and for a practical case wherein μ is reasonably large, one has the criticality relation

$$p_{0_{cr}}^2 = 2\left(\frac{h}{a}\right)^3\left(\frac{E\gamma_a}{a}\right)\left[1 - 4\frac{h}{a}\sqrt[4]{\frac{(1 + \nu')(1 - 2\nu')}{3(1 - \nu')(1 - \nu^2)}\frac{E/h}{E'/h'}} + \ldots\right] \tag{4}$$

which illustrates the practical point that the stiffer the adhesive layer, $k \sim \mu^{1/4} \to \infty$, i.e., higher modulus or thinner layer, the higher the fracture strength.

Similar results have been deduced for a circular blister,[1,3] in contrast to this pressurized strip which is more difficult to test experimentallv.

BIMATERIAL STRIP

In many design situations it is desired to bond two materials of different thickness through an adhesive interlayer and inquire as to the strength of the joint. As an illustration of how such an analysis might be conducted, consider an extension of the example previously discussed (Figure 1). This three-layer medium is assumed to contain a (plain strain) strip flaw of extent $2a$, with the third (bottom) material having its geometry and properties denoted by barred quantities. The only change compared to the previous configuration is that the adhesive layer thickness is to be denoted by $2h'$ so that when the strip rigidity above and below the bond layer are the same, i.e., $E = \bar{E}$, etc., then the previous, rigid substrate, solution symmetric about its mid-plane is properly recovered. For this three-layer media we would write

$$k = \frac{(1 - \nu')(E'/h')}{2(1 - 2\nu')(1 + \nu')} \quad \text{(three-layer strip)} \tag{3a}$$

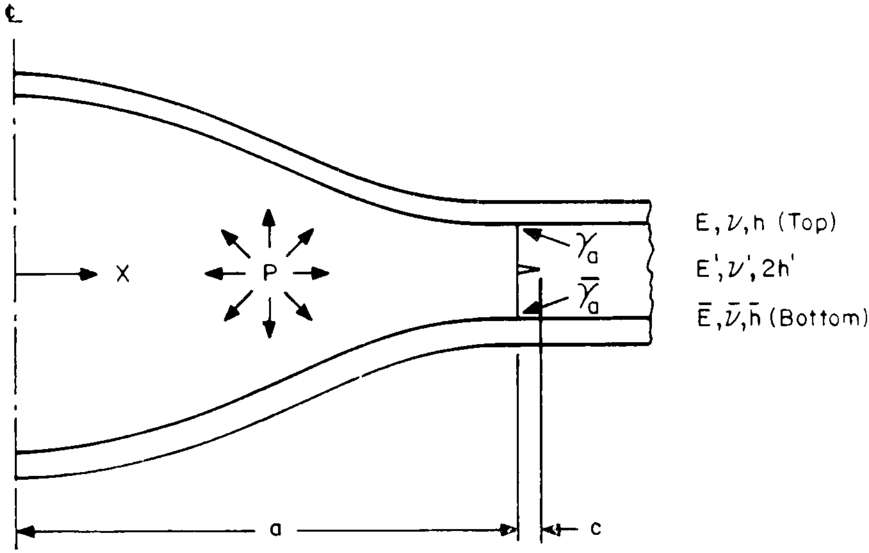

FIGURE 1 Geometry of the three-layer medium.

It is further assumed that the specific adhesive fracture energies between the top layer and adhesive layer (γ_a), and the bottom layer and adhesive layer ($\bar{\gamma}_a$) have been already determined, probably using a pressurized blister test;[1,3] furthermore, that the specific cohesive fracture energy in the adhesive material (γ_c) has likewise been measured, probably using a (plain stress) centrally cracked sheet specimen.[4] For the time being, time-temperature dependence of the fracture energies and moduli are ignored, and only elastic analysis will be conducted.

Because only deflections normal to the layers are considered in this analysis, and particularly because of this assumption with regard to such deformations in the Winkler formulation, the former analysis may be adopted essentially *in toto*, except to modify the top deflection solution (w) to accommodate the bottom deflection solution ($\bar{w}$) and redefining the foundation modulus, i.e., using $2h'$, in order to obtain solutions in the three layers from the governing plate equations.

$$D\, d^4w/dx^4 = p \qquad |x| < a \tag{5a}$$

$$\bar{D}\, d^4\bar{w}/dx^4 = -p \qquad |x| < a \tag{5b}$$

$$D(d^4w/dx^4) + kw = 0 \qquad |x| > a \tag{6a}$$

$$\bar{D}(d^4\bar{w}/dx^4) + kw = 0 \qquad |x| > a \tag{6b}$$

such that after matching the solutions at $x = a$, one finds

$$w(x) = \frac{pa^4 \exp[-\lambda(x-a)]}{12\mu^3(1+\mu)D}\{[(2\mu^2+6\mu+3)\cos\mu + (2\mu^2-3)\sin\mu]\cos\lambda x + [(2\mu^2+6\mu+3)\sin\mu - (2\mu^2-3)\cos\mu]\sin\lambda x\} \quad (7)$$

and a similar expression for $\bar{w}(x)$ by replacing λ, μ, and D by their barred equivalents $\bar{\lambda}$, $\bar{\mu}$ and $\bar{D}$. The normal stress distribution in the interlayer, $f(x)$, is determined from

$$f(x) = k[\bar{w}(x) + w(x)] \quad (8)$$

Fracture criticality is again deduced from the strain energy stored in the system (U), which from Clapeyron's Theorem[5] is one-half the work done by the applied stresses acting through the equilibrium displacement, viz.,

$$U = \frac{1}{2}\int_{-a}^{a} pw(x)\,dx + \frac{1}{2}\int_{-a}^{a}(-p)\bar{w}(x)\,dx = \frac{p^2a^2}{45D}\left\{1 + \frac{5}{4}\frac{4\mu^3+12\mu^2+18\mu+9}{\mu^3(1+\mu)}\right\} + \frac{pa^5}{45\bar{D}}\left\{1 + \frac{5}{4}\frac{4\bar{\mu}^3+12\bar{\mu}^2+18\bar{\mu}+9}{\bar{\mu}^3(1+\bar{\mu})}\right\} \quad (9)$$

ADHESIVE FAILURE

Now the change in work done in creating new surface area, $d\Gamma$, depends upon where the new surface area is created. In the previous problem of a one-sided strip, it was assumed that the fracture initiated simultaneously at both ends of the bond. Thus one had

$$d\Gamma = d(2a\gamma_a) \quad (10)$$

In this case where there are four locations of potential unbonding, it might appear that the corresponding expression would be

$$d\Gamma = d[2a\gamma_a + 2a\bar{\gamma}_a] \quad (11)$$

On the other hand, it seems unlikely that fracture would originate simultaneously at all four corners, especially if the adhesive fracture energies γ_a and $\bar{\gamma}_a$ are different as they should be at the interface of different material combinations. Hence, we choose to use either one, but not both simultaneously, of

$$d\Gamma/da = 2\gamma_a \quad \text{or} \quad d\bar{\Gamma}/da = 2\bar{\gamma}_a \tag{12}$$

as the fracture energy contribution to balance dU/da from (9). It follows of course that this decision implies that in the case of identical materials above and below the bond, the fracture criterion will be affected. In this situation, if the four corners did fail simultaneously, one would deduce

$$d\Gamma/da = 4\gamma_a^* = 2[2\gamma_a^*]$$

and in the ensuing criticality expressions based on (12) one would have to double the fracture energy which is tantamount to increasing the predicted fracture stress by $\sqrt{2}$. This matter must be resolved by experiment but considering normal material variations, it seems unlikely that all four of these corners would fail at the same instant.

As a supplementary remark one could object on the bais of consistency to using the factor two in (10). Why must this one flaw fail at both its ends simultaneously? To some extent the position is indefensible even though Griffith,[6] Sneddon,[7] and Williams[8] have adopted simultaneous extension in the one-, two- and three-dimensional plate, penny-shape and spherical configurations, respectively. To a large extent experimental results seem consistent in supporting the dual extension theory. Nevertheless, for the adhesive debonding situation, (pending experimental results) four-point initiation will be precluded, and leads to conservative criticality predictions by equating $dU/da = d\Gamma/da$.

Continuing the analysis therefore, one finds for debonding at the top layer interface,

$$p_{cr}^2 = \frac{2\left(\frac{h}{a}\right)^3\left(\frac{E\gamma_a}{a}\right)}{1 + \phi(\mu)} \cdot \frac{1}{1 + \dfrac{D[1 + \phi(\bar{\mu})]}{\bar{D}[1 + \phi(\mu)]}} \tag{13}$$

or debonding at the bottom layer interface,

$$\bar{p}_{cr}^2 = \frac{2\left(\frac{\bar{h}}{a}\right)^3\left(\frac{\bar{E}\bar{\gamma}_a}{a}\right)}{1 + \phi(\bar{\mu})} \cdot \frac{1}{1 + \dfrac{\bar{D}[1 + \phi(\mu)]}{D[1 + \phi(\bar{\mu})]}} \tag{14}$$

where $\phi(\mu)$ is defined implicity in (1), and the lower of p_{cr} and $\bar{p}_{cr}$, i.e., p_{min}, give the criticality estimate and location.

$$p_{min} = \text{minimum}\,[p_{cr}, \bar{p}_{cr}] \tag{15}$$

COHESIVE FAILURE

In some situations, however, it may happen that failure will not take place at the interface at all, but rather that there will be cohesive failure in the adhesive itself. Certainly there is a strong, biaxial stress distribution within the adhesive and if a flaw exists within or at the edge of the adhesive, it presumably would initiate the failure in this region preferentially. It would correspond to relatively high values of the adhesive energy compared to the cohesive value. Without attempting to solve for the rather complicated cohesive fracture criticality condition in the adhesive at this time, consider a simple approximate solution which demonstrates the major features of the analysis.

The approximate stress in the adhesive is given from (8), which, it is to be emphasized, neglects any shear stress. At the edge of the adhesive, $x = a$, one has

$$f(a) = p\left\{\frac{\mu}{3(1+\mu)}[2\mu^2 + 6\mu + 3] + \frac{\bar{\mu}}{3(1+\bar{\mu})}[2\bar{\mu}^2 + 6\bar{\mu} + 3]\right\} \quad (16)$$

Assuming for the moment that this stress is uniformly distributed, instead of having its characteristic damped oscillatory behavior, or some other averaged value, one may inquire as to the criticality threshold for a flaw in the adhesive layer subjected to the (conservative) tensile stress $f(a)$. For example, if there was a small edge crack, of depth c, then we find that it will extend when $f(a)$ reaches the (plane strain-incompressible) critical value of

$$f(a)_{cr} \doteq \sqrt{\frac{2}{3}\frac{E'\gamma_c'}{c}} \quad (17)$$

which depends upon the modulus (E') and specific cohesive fracture energy (γ_a') of the adhesive.

CONCLUSION

Cohesive or adhesive failure in the three-layer medium will therefore depend upon the relative magnitudes of (13), (14) and (17). Explicitly, cohesive failure due to a small edge crack will result if

$$\sqrt{\frac{2}{3}\frac{E'\gamma_c'}{c}} < p_{\min}\left\{\frac{\mu}{3(1+\mu)}[2\mu^2 + 6\mu + 3] + \frac{\bar{\mu}}{3(1+\bar{\mu})}[2\bar{\mu}^2 + 6\bar{\mu} + 3]\right\} \quad (18)$$

If the inequality is reversed, adhesive failure will occur at $p_{\min}$, the lower of (13) and (14), which also will indicate the location of the interfacial debonds.

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